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Space truss bridge optimization by dynamic programming and linear programming

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ABSTRACT: The objective of the current study is two-fold. First, minimize the cost of a bridge by determining the optimal number and location of the piers. Second, minimize the weight, size and cost of the truss for the individual spans determined in the prior step. There are three main categories invoke to optimize a truss structure, namely, sizing, shape and topology optimization. The aim of the current study is to optimize the topology of a space truss while maintaining the external force is balanced in all considered degree of freedoms and meeting up the Euler buckling and material strength satisfactory. Two cutting-edge optimization techniques, namely, dynamic programming and linear programming were employed in this study. In one hand, dynamic programming is an approach for making a sequence of decisions in an optimal way for a given recursive problem. In dynamic programming, a problem is generally divided into stages that give the best outcome based on the previous decision. On the other hand, linear programming is a method for optimizing a scenario that can be described mathematically by linear relationships. Results showed that the adopted strategy can determine the optimal bridge configuration both in small and large scale very well.

1 INTRODUCTION

Optimization is a technique used to select the best option from available alternatives, subject to certain conditions. There are many different programming methods used to optimize a variety of problems. Two that were used in this study are dynamic programming (DP) and linear programming (LP). DP is an approach for making a sequence of decisions in an optimal way. At a basic level it is taking a small part of a problem, finding an optimal outcome for that small part, expanding the problem by a small amount, and solving again, until the expanded problem encompasses the original problem (Sniedovich 2010). By then tracing back the optimal decision taken at each step, the optimal decision for the whole can be found. In dynamic programming, a problem is generally divided into stages. Stages can be thought of as a new small problem to be solved that builds on the previous solution. Each stage then has a number of states, decisions and decision updates.

On the other hand, LP is a method for optimizing a scenario that can be described mathematically by linear relationships. Many problems can be formulated and solved in this style of programming. One example is truss optimization, which utilizes LP, or in some cases network flow programming, to optimize weight or size or cost of the truss structure. LP is the most successful and most often used technique for solving truss problem because of its system of equations deal with member dimensions those bound to linear domain (Li et al. 2009; Rajeev & Krishnamoorthy 1992; & Rahami et al. 2008). The objective of this study is two-fold. One is to minimize the cost of a bridge by determining the optimal location of the piers. Second is to minimize the weight of the truss for individual spans determined in the previous step.

2 BRIDGE SPAN OPTIMIZATION

2.1 *Dynamic Programming- Overview*

Dynamic Programming is an approach for optimizing multistage decision processes. It is based on Bellman's Principle of Optimality: '*an optimal policy has the property that, regardless of the decisions taken to enter a particular state in a particular stage, the remaining decisions must constitute an optimal policy for leaving that state*' (Sniedovich 2010).

A multistage decision process is a process that can be separated into a number of sequential steps, or stages, which may be completed in one or more ways. The options for completing the stages are called decisions. A policy is a sequence of decisions, one for each stage of the process. The condition of the process at a given stage is called the state at that stage; each decision effects a transition from the current state to a state associated with the next stage. It is to be noted that a multistage decision process is finite if there are only a finite number of stages in the process and a finite number of states associated with each stage (Sniedovich 2010). Multistage decision processes have returns associated with each decision which vary with stages and states. The objective in analyzing such decision processes is to determine an optimal policy, one that results in the best total return. Thus, DP is a method to solve optimization problem containing a specific objective.

2.2 *Context of the Present Study*

The context of the DP part for this study is to design a bridge in terms of number of piers and pier spacing which minimizes the total cost. The bridge span was considered as a 1250 m length and the bedrock profile across the ravine at the bridge site is assumed from a river bed profile found in GoogleEarth that was located over Narayanganj, Bangladesh.

The model includes both cost constraints and spatial constraints. The spatial constraints are: the bridge may have not more than 5 piers and no individual span may exceed 500 m. The cost constraints ensure that a minimum cost configuration would be chosen. Simplified cost estimating formulae are available for individual spans of decks and for piers. The cost of a single span is assumed proportional to the square of the span and is given by:

$$\text{Cost of deck span} = \text{DConst}_1 + \text{DConst}_2 * (\text{span length})^2 \qquad (1)$$

where DConst_1 and DConst_2 are given constants as assumed a value of 20000 and 2, respectively. The values of these constants are considered based on rational data available in literature.

The cost of a single pier is assumed proportional to its height and is given by:

$$\text{Cost of pier} = \text{PConst}_1 + \text{PConst}_2 * (\text{pier height}) \qquad (2)$$

where PConst_1 and PConst_2 are given constants as assumed a value of 50000 and 11000, respectively. Akin to earlier case, the values of these constants are considered based on rational data available in literature.

2.3 *Model Assumptions*

The assumptions made during the problem formulation are largely present in the cost functions. The cost coefficients determine how the model chooses the most economical pier locations because of the weights assigned to pier depth and span length. Changing these would have been a significant impact on the result obtained. The choice of limit for the span length to 500 m is another scope of the study, ergo a shorter maximum length would end up with a different pier configuration result.

2.4 *Define Stages and Stage Numbering*

A stage was consisted of one deck span and the supporting pier at the left hand (LH) end of this span. Stage numbering was considered from left to right with the left hand abutment included in stage 1 and the right hand (RH) abutment included in stage 7 (Fig. 1a).

2.5 *Define States*

State for a stage was considered a positive center line location for a pier and was noted from RH abutment. In this way, an interval of 50 m was assumed between discrete state values. For example, State 1 would correspond to the location of the RH abutment. State 21 (at 1000 m from RH) will correspond to the location of the LH abutment (Fig. 1a).

2.6 *Define Decision Variable*

At a particular stage and state, i.e. for a given pier and center line location, the decision choice would be the length of span to the next pier to the right (Fig. 1a).

2.7 *State Transformation Equation*

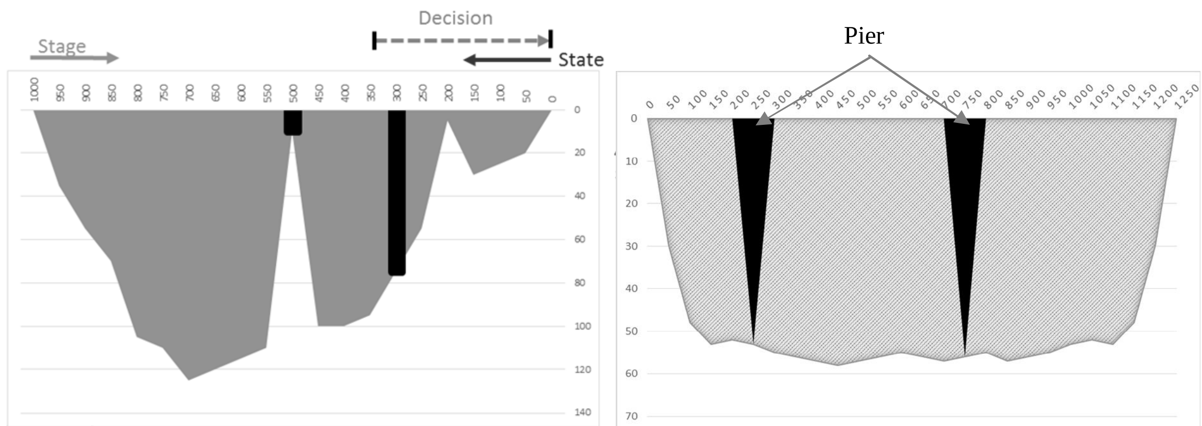
Given a state and a decision (i.e. span to the next pier to the right), state transformation equation would be the span length resulting from the difference between state (section 2.5) and decision variable (section 2.6).

2.8 *Stage Return Function*

One stage cost will be the sum of the pier cost and deck cost. The minimum cost in a state would be the decision of that particular stage.

2.9 *Recursion Equation*

The same formulation is adopted for all of the stages started from Stage 7 to Stage 1.



a) (b)

Figure 1. a) Problem definition and optimal locations for piers according to the DP model, b) DP optimized pier location

2.10 *DP Result*

The dynamic programming model yielded a two-pier bridge as the optimal result, with the piers located at x = 250 m and x = 750 m (Figure 1b). Given the profile of the river bed, there are no obviously shallow locations to place piers that minimize pier height, so the result has been dominated by span length and an attempt to have as few piers as possible. This meant the optimal spans measured 300 m, 200 m and 500 m, again from right to left. The decision makes sense, because the deeper sections of the river are associated with higher costs because of pier height. The model has therefore chosen to place one pier at the maximum possible span length to avoid having piers in the deepest part of the river and then has chosen a balance between span cost and pier cost to place the second pier.

3 SPACE TRUSS OPTIMIZATION

There are three main categories invoke to optimize a truss structure:

i. Shape optimization (variables are nodal coordinates)
ii. Sizing optimization (variables are cross-sectional areas of the members) and
iii. Topology optimization (variables are the location of links in which connect nodes).

The aim of this study is to do topology optimization. In this section the application of LP for optimization of space truss has been discussed. A generalized model which could be extended to any configuration has been modelled in a programming language, namely, AMPL (Applied Mathematical Programming Language). The model set up was first validated for a simple truss configuration. This was later extended to optimize a large scale truss problem.

A structure is called to be a space truss if it is (Hibbeler 1998 & Popov 1998):

- externally (geometric) stable, and
- has $(3j-r)$ members [j and r are number of joints and support reactions, respectively].

3.1 *LP Problem Formulation*

Then definition of the structural analysis problem to solve the truss structure by LP is described as follows (Ghasemi et al. 1997; Li et al. 2009; Rajeev & Krishnamoorthy 1992; Rahami et al. 2008; & Rasmussen & Stolpe 2008).

t_{ij} = tension in member $\{i, j\}$ in domain set A

$t_{ij} = t_{ji}$; Compresion = - Tension

$p_{i(X,Y,Z)}$ = position vector for joint i

$f_{i(X,Y,Z)}$ = external force vector for joint i

$$l_{ij} = \text{length vector for member}\{i, j\} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} \quad (3)$$

$$u_{ij(X,Y,Z)} = \text{unit vector for member}\{i, j\} = \frac{p_{j(X,Y,Z)} - p_{i(X,Y,Z)}}{\left\| p_{j(X,Y,Z)} - p_{i(X,Y,Z)} \right\|} \quad (4)$$

$$u_{ji(X,Y,Z)} = -u_{ij(X,Y,Z)} \quad (5)$$

$$\text{minimize} \sum_{\{i,j\}\in A} l_{ij} \left| t_{ij} \right| \quad (6)$$

$$\text{subjected to} \sum_{\substack{j:\\ \{i,j\}\in A}} u_{ij(X,Y,Z)} t_{ij} = -f_{i(X,Y,Z)} \; ; \quad i = 1,2,.....,n \quad (7)$$

Reformulated LP formulation:

$$t_{ij} = t_{ij}^{+} - t_{ij}^{-} \; ; \quad t_{ij}^{+}, t_{ij}^{-} \geq 0 \quad (8)$$

$$\left| t_{ij} \right| = t_{ij}^{+} + t_{ij}^{-} \quad (9)$$

$$\text{minimize} \sum_{\{i,j\}\in A} \left(l_{ij} t_{ij}^{+} + l_{ij} t_{ij}^{-} \right) \quad (10)$$

$$\text{subjected to} \sum_{\substack{j:\\ \{i,j\}\in A}} \left(u_{ij(X,Y,Z)} t_{ij}^{+} - u_{ij(X,Y,Z)} t_{ij}^{-} \right) = -f_{i(X,Y,Z)} \; ; \quad i = 1,2,.....,n \quad (11)$$

$$\text{Buckling load for member}\{i, j\},\ P_{critical,ij} = \frac{\pi^2 EI}{\left(K_e l_{ij}\right)^2} \; ; \; I = \text{Moment of inertia and } K_e = 1 \quad (12)$$

3.2 *Example Problem*

Working of the model has been discussed in this section with the help of a simple configuration.

i. A grid system was considered as illustrated in Figure 2a. There are 5 nodes starting from 0 to 4 along *X* direction and 3 nodes starting from 0 to 2 along *Y* and *Z* direction. Each node is equally spaced at a distance of 1m apart in *X* and *Y* directions and 2 m apart in *Z* direction.
ii. A load of 50 kN is applied at point *P (3, 1, 0)*. The configuration is arrived on such that the truss has fixed supported at nodes of *(0, 0, 0)*, pinned connection at *(0, 0, 2)* and roller support *(0, 2, 2)*.

iii. The material properties of the members are considered as follows: modulus of elasticity, $E=2x10^5$ N/mm^2, density = 78.89 kg/m^3 and maximum allowable stress =250 N/mm^2.
iv. Arcs are defined such that the nodes are connected in all possible ways (Fig. 2b).
v. No force balance equation was applied at anchored joints.

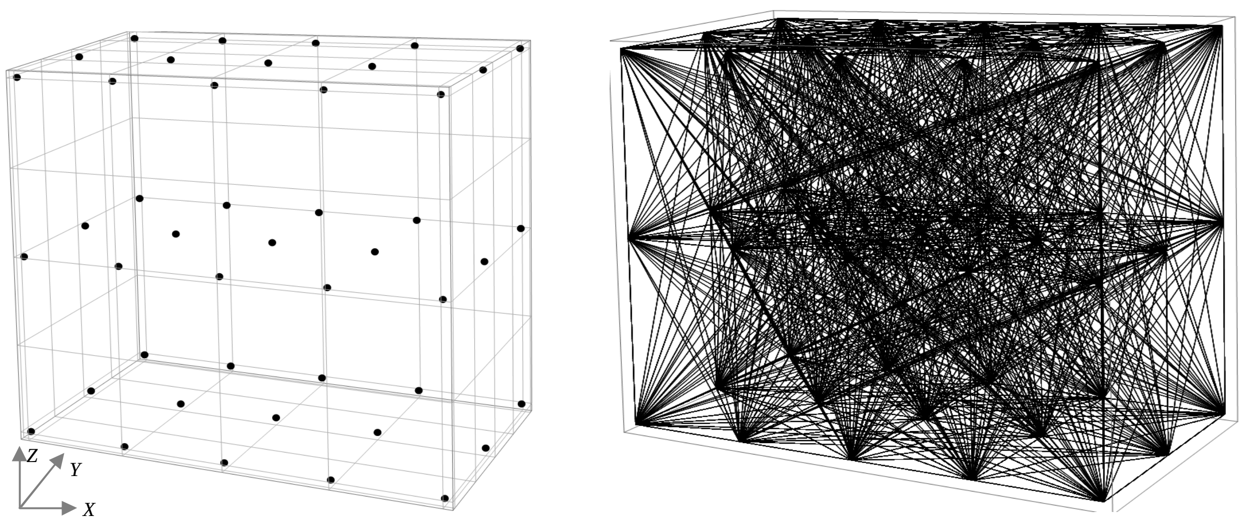
3.3 *Objective Function*

The objective of this LP formulation is to minimize the weight of the structure and arrive on an optimal configuration with the area of cross-section for the members being used (Equation 10). Since the force depends on the area of cross-section, the objective function is defined as minimizing the total absolute force.

3.4 *Constraints*

The constraints for the optimization are

i. Satisfy the equilibrium equation that is sum of forces along *X* direction at every node should be zero (Equation 11). Similarly the sum of the forces along *Y* and *Z* direction should be zero.
ii. An additional constraint is added such that the critical member size does not go beyond 1000 mm^2 due to have material's physical limit.
iii. Forces of the members would be governed by the stability of member relating to Euler buckling (Equation 12) and strength of the material up to elastic stage.



a) (b)

Figure 2. a) Node definition and b) search domain

3.5 *LP Result*

Figure 3 shows the structure that is obtained after optimization algorithm runs. However, the member force is not showing here due to have space limitation. Later on, in order to verify the algorithm result, the optimized structure was evaluated by 'section cut' method and got the acceptable agreement.

3.6 *Model Assumptions*

i. Decision variables that is the cross section are of the truss members are continuous.

3.7 *Pros of the Model*

i. The model is simple and easy to use. The user is required to specify the coordinates for load and support conditions.
ii. The model is capable to handle complex and large structural problems without losing accuracy and/or demanding more computational power.

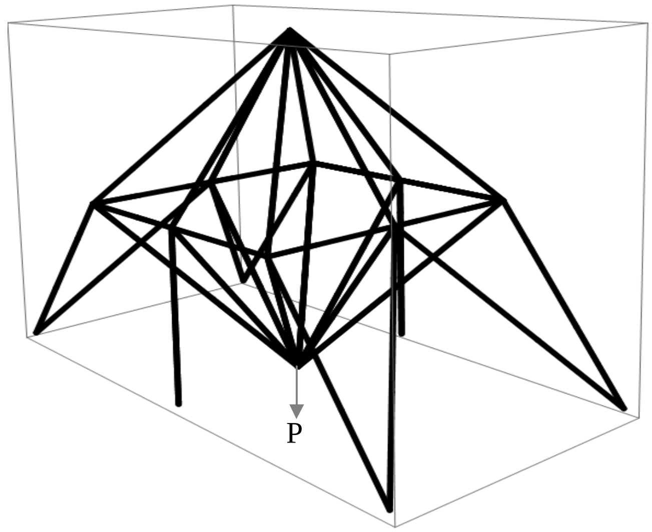


Figure 3. Optimal truss configuration within nodes boundary (5 x 3 x 3)

3.8 *Cons of the Model*

i. Since the cross-sections are considered as continuous, the model might not the precise representation for a real case scenario.
ii. The truss design problem that is formulated in the present study presumes that the truss structure itself is not affected by its own weight.

4 CONCLUSIONS

The study was investigated the optimum number of piers and pier spacing which minimizes the total cost of a bridge construction. The dynamic programming model was yielded a two-pier bridge as the optimal result. In DP formulation the cost function for piers considers only the height. A more realistic cost function would have a term relating span length to pier diameter and consequently would effect on cost behavior. As a follow-up step, the application of linear programming for optimization of space truss suited for the span length determined by DP has been discussed in this study. A generalized LP model which could be extended to any configuration has been modelled in a programming language, namely, AMPL. Results showed that the adopted strategy can determine the optimal bridge configuration both in small and large scale very efficiently in terms of computational cost and accuracy.

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