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Dynamic modelling of railway bridge train interaction for fatigue damage assessment

A.K. Rahman, B. Imam & Y. Wang
Dept. of Civil & Environmental Engineering, University of Surrey, Guildford, United Kingdom

ABSTRACT: The estimation of fatigue life is becoming a key issue for ageing railway bridge asset owners with regards to prioritizing maintenance, rehabilitation and replacement of existing structures. With increased volume of rail traffic, axle loads, train speeds, different locomotive/wagon configurations and the potential effects of resonance, understanding better the rate at which fatigue damage is being accumulated is of significant interest due to the complex nature and interaction of these loading phenomena with the bridge. Using the Euler-Bernoulli beam formulation for representing the bridge subjected to a series of moving loads, this paper presents a mathematical model which can be used to capture dynamic effects on fatigue damage accumulation. The model is demonstrated by assessing the dynamic response of a single span plate girder type bridge for different train loadings. The dynamic response from the analytical model is compared with dynamic amplification factors suggested by fatigue assessment codes. The results presented in this study show that fatigue damage can be significantly affected by the dynamic interaction between the trains and bridge. The model presented in this study can be used to quickly and efficiently gain key structural and fatigue life information to help in decision making for asset owners.

1 INTRODUCTION

1.1 *Background*

Many railway networks throughout the world are having to support significant increases in capacity, raising serious structural concerns for ageing bridges. For all operational bridges there is an obligation for the asset owner to keep them structurally safe and in service as long as possible due to their economic and social function. The costs associated with the structural maintenance and replacement must be assessed taking into account the fatigue life of the bridge and potential risks of a structural failure. The prioritization of maintenance, and ultimately the replacement of these bridges, is now becoming an area of significant interest and activity. Part of this activity requires the estimation of the fatigue life of the bridge. Fatigue damage is accumulated on bridge structural members and connection due to the cyclic passage of trains. Reliable estimation of the fatigue life of a bridge requires knowledge of the damage already accumulated, which for many ageing bridges can be difficult to determine, and future damage from modern trains.

Fatigue damage accumulation on a bridge is affected by the number and magnitude of the cyclic stress ranges induced by train passages. However, with increased axle loads, volume of traffic and train speeds the rate at which damage is being accumulated has increased. The problem is further exacerbated by the dynamic effects of the train-bridge interaction, a problem that is more pronounced for high speed trains, as this may induce resonance effects and greater dynamic impact for the bridge. Current bridge design codes consider dynamic effects by scaling the results of a static analysis to envelope dynamic effects with a parameter known as the Dynamic Amplification Factor (DAF). The expressions that calculate DAFs were originally obtained by empirical means from the results of field tests on specific bridges under a range of velocities, providing global DAF values that are functions of the bridge/member span. Developing analytical models capable of accurately predicting DAFs will allow for a wider range of bridge types and configurations as well as a wide range of train velocities and train axle configurations to be captured. Previous studies carried out on truss bridges, in the form of detailed dynamic analyses, showed that the code-defined DAFs may be overly conservative in some cases whilst in other cases it may underestimate the dynamic effects (Imam & Yahya, 2014). Without an accurate estimation of the DAF any fatigue calculations may be subject to uncertainties due to their sensitivity to member stress levels. An overestimation may lead to operational safety concerns whilst an underestimation could mean untimely maintenance, replacement and expenditure for the asset owner. With the railway industries under challenging financial and budgetary constraints, as well as the complications of the management of the rail networks, there is now greater emphasis placed upon engineers to be able to accurately assess and establish the remaining fatigue life of bridges.

The aim of this paper is to address the shortcomings in the current bridge design/assessment codes by developing an analytical model by which fatigue damage can be estimated by explicitly taking into account the dynamic response of the bridge and the different train configurations and velocities that may operate on a network.

2 MOVING LOAD MODEL & DYNAMIC AMPLIFICATION

2.1 *Historical Development of the Moving Load Model*

The first investigations of railway bridge vibrations under moving loads date back to the middle of the 19^{th}century (Garg & Dukkipati, 1984). Contributions by Willis (1849), Stokes (1867) and Robinson (1887) on the dynamic response of bridges as a result of a moving load were amongst the first which paved the way for later engineers to investigate this phenomenon in more detail. It was the collapse of Dee Bridge in Chester (UK) in 1847 that led to Willis and Stokes to focus on the investigation of the dynamic stresses in beams under moving loads (Karnovsky, 2012). The moving load problem was formulated by Willis and later solved by Stokes in 1849. The basic formulation of the problem on which the early investigations were based upon were on the moving load and moving mass models. These models were subsequently extended to include the inertia of the beam which represented the bridge. The difficultly of solving these models mathematically increases when the inertia of the force and the beam are considered.

The moving force model is a relatively straight forward computation and this formulation was solved by Winkler and Mohr independently in 1868 (Karnovsky, 2012). The moving mass model, known as the Willis-Stokes problem, was first formulated by Willis in 1849 and subsequently solved by Stokes. By neglecting the beam's mass, Willis (1849) arrived at a fourth order partial differential equation representation of the problem. During the period of 1900-1940, railway bridge dynamic investigations mainly focused on developing analytical and approximate solutions to simplified dynamic problems. Prominent researchers investigating vibrations associated with moving loads during this time included Jeffcott (1929), Lowan (1935) and Inglis (1934) who provided a general treatment on the dynamics of railway bridges. Inglis used the method of harmonic analysis for assessing the dynamic response of railway bridges due to the distributions of moving loads in the form of concentrated forces, sprung and un-sprung masses and harmonic forces acting on a beam.

With the development of digital computers from the 1940's more complex and realistic models, comprising both bridge and trains, started to be formulated by researchers (Fryba, 1999). Fryba made significant contributions in the subject area of vibration of solids under moving loads. He provided a comprehensive theoretical formulation of the moving load problem acting on elastic and inelastic solids which included beams during the 1960's. The moving load problem was again reviewed by Fryba in the 1970s and 1990s and enabled other researchers in the field, most notably Yang et al (2004), to apply and expand on Fryba's closed form analytical formulations in understanding railway bridge dynamic problems. Yang, principally focusing on high-speed railway bridges, provides a broad and systematic assessment on the problem of moving loads but also including the train interaction dynamics with the bridge. The works of Fryba are provided in his book which entirely deals with the subject of moving loads (Fryba, 1999). The mathematical model of the moving load problem by Fryba (1999) is utilised in this work and extended to present a more comprehensive model that accounts for different bridge and train configurations as well as incorporating a fatigue damage model.

2.2 *Evolution of the Dynamic Amplification Factor*

The dynamic amplification factor (DAF) is defined as the ratio between the dynamic and static responses. In the available literature, the DAF is referred to by a number of other terms such as impact factor, impact coefficient and dynamic magnification factor. Estimation of the DAF depends on a number of parameters, which includes; the span, self-weight of the bridge and train speed amongst others. Many early bridge design codes empirically relate the DAF to a single parameter of the bridge, either the span of the bridge or its resonant frequency. This was the basis by which a global dynamic amplification factor (DAF) was calculated.

Three of the most significant works on the investigation of bridge impacts due to the interaction of trains were performed in the early part of the 20th century (Looney, 1944). These were considered to be the first investigations which specifically studied impact for the purposes of introducing impact allowance factors in the bridge design codes. These investigations also paved the way for other future investigations on the subject. The first study was performed in the US by the sub-committee of the American Engineering Association who were tasked with the investigation of the various factors which contributed to impact. The study was based on a series of field tests measuring deflections on bridges using trains running at different speeds. This study culminated in the establishment of the empirical formulas for calculating impact allowance factors that were introduced into the American bridge design codes.

2.3 *Current Bridge Design Code Dynamic Amplification*

The calculation of DAFs is specified for fatigue limit states in the Network Rail (2006) assessment codes for the assessment of bridges in the UK. The calculation method given in the Network Rail assessment code, adopted in this study, provides a distinction between longitudinal and transverse members. For fatigue calculations the dynamic increment, $\varphi$for bending of a longitudinal member is given by the same equations as given in Eurocode 1, except for the calculation of the parameter *k*for the basic dynamic increment, ϕ' and the increment for the track irregularity, ϕ''. The procedure for calculating the DAF is as follows:

$$k = \frac{v}{4.47 L_{\Phi} \eta_o} \tag{1}$$

The basic dynamic increment, ϕ' and increment for track irregularity, ϕ'' are given by:

$$\phi' = \frac{k}{1-k+k^4} \quad and \quad \phi'' = \alpha\left[56e^{-\left(\frac{L_\Phi}{10}\right)^2} + 50\left(\frac{L\eta_o}{80} - 1\right)e^{-\left(\frac{L_\Phi}{20}\right)^2}\right] \quad but\ > 0 \tag{2}$$

Where *α=0.002v but not >0.01* and *v* is the train speed in mph, which is normally taken as the permissible speed for the bridge. The determinant length is given by L_Φ and L is the span of the bridge member (centre-to-centre of supports) in metres and η_o is the fundamental natural frequency of vibration in Hertz of the structural member based on δ_o resulting from the uniformly distributed self-weight deflection, *w*.

$$\eta_o = \frac{17.75}{\sqrt{\delta_o}} \quad where\ \delta_o\ is\ in\ mm\ given\ by\ \delta_o = 1000\frac{5wL^4}{384EI} \tag{3}$$

In equation (3) *EI* is the flexural rigidity of the bridge. The upper and lower bounds of the natural frequency η_o are estimated as follows with the DAF calculation for bending given by equation (5).

$$\eta_o = 94.76L^{-0.748} \quad (upper\ bound) \tag{4}$$

$$DAF_{bending} = 1 + 0.5\left(\phi' + \frac{\phi''}{2}\right) \tag{5}$$

Using the above methodology, the dynamic amplification for a bridge span of 18.1m is plotted against the train speed in Figure 1. The average train speed on UK networks is around 95-105km/h but passenger trains have now been introduced with speeds of up to 201km/h on various mainlines(Gaillard, 2003). Based on the DAF curve of Figure 1 for a bridge span of 18.1m the amplifications can vary from 1.065 at 50km/h to 1.143 at 105km/h and to 1.333 at 201km/hr.

3 STATIC AND DYNAMICMOVING LOAD MODELS

3.1 *Introduction*

The representation of the train bridge interaction as a series of moving loads, as depicted in Figure 2, represents the main challenge in the formulation of the analytical model for both the static and dynamic cases. In the static case, the inertia of the beam is neglected and therefore the response of the beam is not accounted for in the calculation of the deflection, bending moment and stresses. The analytical model presented in this paperis created using MATLAB.

In both the static and dynamic analysis, the axle loads and the position of each axle are defined in terms of two row vectors as follows.

$$F_D = [\,F_1 F_2 \ldots\ldots\ldots\ldots\ldots F_n] \quad \text{and} \quad X_D = [\,X_1 X_2 \ldots\ldots\ldots\ldots\ldots X_n] \tag{6}$$

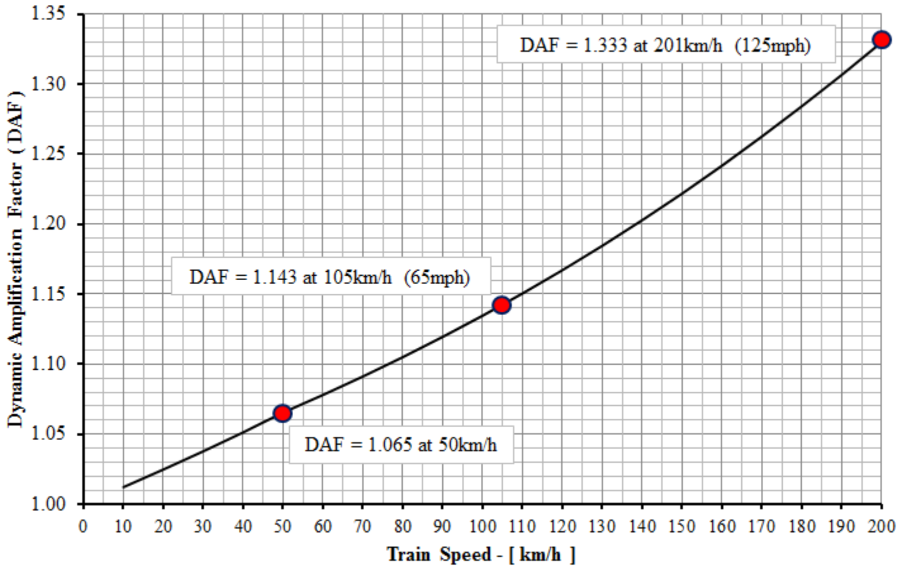

Figure 1. Dynamic amplification factor (DAF) based on Network Rail Code (2006).

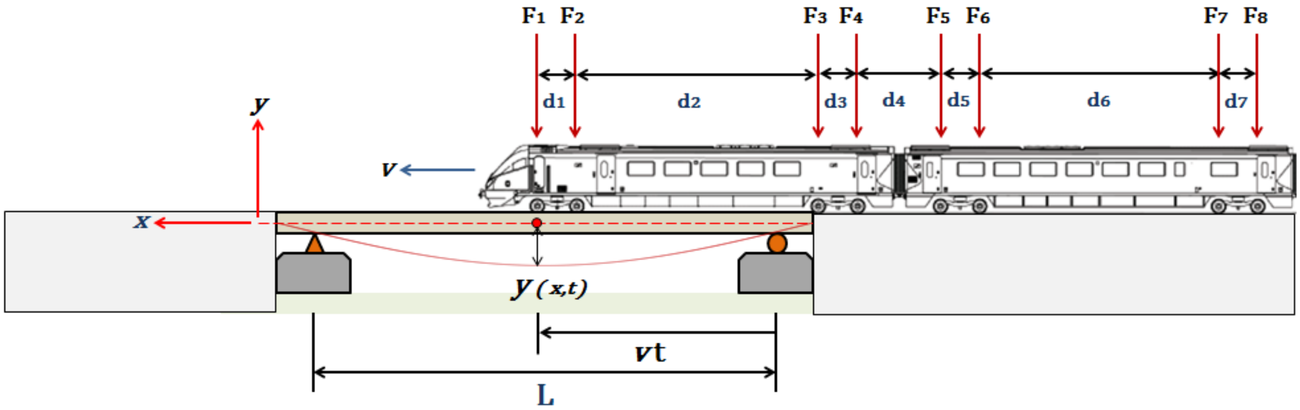

Figure 2. Bridge rain interaction model.

The distance X is defined from the first axle which is placed at position$x=0$ on the bridge. A time increment, T_{inc} is defined for the analysis. The incremental marching step for the train is defined using the velocity of the train in m/s, $L_{inc}= vT_{inc}$. For the static analysis the bridge span is divided into equal increments of L_{inc} giving the following incremental positions on the bridge.

$$X_{br} = [\, x_1 x_2 \dots \dots \dots \dots \dots x_n] \quad (7)$$

The analytical model requires the distances of each axle to be equally spaced. This is achieved by introducing dummy axle distances at increments of L_{inc} between the existing defined axles. An axle force of zero is assigned at these dummy axle positions. As this has introduced additional axle loads and distances new position and load row vectors are thus defined. The following other parameters are defined for the analytical model.

$X_{tot} = X_n + L$	*Total length of train plus bridge*	(8)
$t_{cross} = \frac{X_{tot}}{v}$	*Total time to cross bridge by train*	(9)
$t = [t_1 t_2 \dots \dots \dots \dots t_j]$	*Time array at* T_{inc} *increments where* $t_j = 1.5t_{cross}$	(10)
$t_n = \frac{X_{D,n}}{v}$	*Time when the* N^{th} *force enters the bridge*	(11)
$T_n = \frac{(L+X_{D,n})}{v}$	*Time when the* N^{th} *force leaves the bridge*	(12)
$x_n = vt - X_{D,n}$	*The position of the* n^{th} *axial force,* $F_{D,n}$	(13)
$T_n = \frac{(L+X_{D,n})}{v}$	*The departure time of the nth axial force,* $F_{D,n}$	(14)

The position of the first axial at $x=0$ (reference point at the entry point of the bridge) is $X_{D,1}=0$. The above formulations are based on the axles being equi-distance apart with dummy loads. This method facilitates the assessment of real trains which may have varying axle distances.

3.2 *Moving Load Model Based on Classical Beam Bending Theory*

For comparison purposes the deflection and stresses are calculated for a bridge main longitudinal beam based on a static analysis using classical beam equations and employing the principle of superposition. Using this type of analysis, the stresses are then scaled using a dynamic amplification factor, DAF, which is calculated using the Network Rail bridge assessment code, as discussed above, to account for dynamic effects. This is referred to as a quasi-static (q-static) case in this work. The bending moment of the bridge at a point at a distance a from the end of the beam, which in this study is at the position of the mid-span or $0.5L$, is calculated using equation (15) where x is the position of the moving force given by $x=vt$ and P is a unit load. The equation effectively creates the bending moment influence lines for the member under consideration.

$$M_{br,x} = \sum_{x=0}^{x=L} \left(\frac{Px(L-a)}{L} \langle x < a \rangle \bigg| \frac{PL}{4} \langle x = a \rangle \bigg| \frac{Pa(L-x)}{L} \langle x > a \rangle \right) \tag{15}$$

For the series of axle loads crossing the bridge the total bending moment is given by equation (16) where N_{XD} is the total number of axles, including dummy axles, and N_{br} is the total bridge span distance increments.

$$M_{static,t} = \sum_{j=1}^{N_{XD}} \sum_{j+1}^{j+N_{br}} F_j M_{br} \tag{16}$$

By applying the DAF the static analysis stress time history is converted to a dynamic stress time history.

$$\sigma_{qstatic,t} = DAF \times \frac{M_{static,t}\bar{y}}{I} \tag{17}$$

where $\bar{y}$ is the distance from the neutral axis to the outer fibre of beam and I is the second moment of area.

3.3 *Bridge Train Interaction Model for Moving Loads Using Euler-Bernoulli Beam Theory*

This analytical formulation of the problem is based on the extended classical Euler-Bernoulli beam model presented by Fryba (1999) for a series of moving loads, given in equation (18).

$$EI\ \frac{\partial^4 y(x,t)}{\partial x^4} + \mu\ \frac{\partial^2 y(x,t)}{\partial t^2} + 2\,\mu\,\omega_d\ \frac{\partial y(x,t)}{\partial t} = \sum_{n=1}^{N} \varepsilon_n(t)\ \delta\ (x - x_n)\ F_n \tag{18}$$

Fryba (1999) provides the closed form solution of equation (18) in the time domain which enables the calculation of the bending moment at a specific location x along the bridge, given by equation (19). The closed form solutions introduces the Heaviside Step Function $H(t)$, equation (20) and the Dirac Delta Function $\delta(x)$ which is a derivative of $H(t)$. The Dirac Delta Function, also known as the 'Unit Impulse Function', $\delta(t)$ is used to model the density of an idealized point mass (single concentrated axle load in this case) as a function that is equal to zero everywhere (where $t \neq 0$) except for zero ($t=0$) where it is infinite, and whose integral over the entire real line is equal to unity (Bracewell, 2000).

$$M(x,t) = \sum_{j=1}^{\infty} \sum_{n=1}^{N} M_0\, F_n j^3 \omega \omega_1^2 \left[\, f\,(t - t_n) H(t - t_n) - (-1)^j\ f\,(t - T_n) H(t - T_n) \right] sin \frac{j\pi x}{L} \tag{19}$$

Where

$$\varepsilon_n(t) = H(t - t_n) - H(t - T_n) \tag{20}$$

$$f(t) = \frac{1}{\omega_j' D} \left[\frac{\omega_j'}{j\ \omega}\ sin(j\omega t + \theta) + e^{-\omega_d t}\ sin\left(\omega_j' t + \varphi\right) \right] \tag{21}$$

In equation (19), M_o is the unit load (1ton) bending moment at the mid-point position of a simply supported bridge given by equation (22).

$$M_0 = \frac{PL}{4} \tag{22}$$

The natural frequencies, ω_i and f_i and the damped vibration frequency, ω_d are given as follows.

$$\omega_j = \frac{j^2 \pi^2}{L^2} \sqrt{\frac{E\,I}{\mu}},\ j = 1,2,3,\dots,\ and\ \ f_j = \frac{\omega_j}{2\pi}, j = 1,2,3,\dots,\ and\ \omega_d = f_1 \vartheta \tag{23}$$

where f_1 is the first fundamental natural frequency of vibration and ϑ is the dimensionless logarithm decrement, which for steel railway bridges is given by $1/(0.3L\text{-}0.0012L^2)$ (Fryba, 1999).The mode shape of the bridge for the j_{th} non-damped vibration mode is given by the term $sin(j\pi x/L)$ in equation (19).Other parameters of the mathematical model include the following (Fryba, 1999).

$$\omega'_j = \sqrt{\omega_j^2 - \omega_d^2} \quad and \quad D = \sqrt{\left(\omega_j^2 - j^2\omega^2\right)^2 + 4j^2\omega^2\omega_d^2} \tag{24}$$

$$\theta = \tan^{-1}\frac{-2j\omega\,\omega_d}{\omega'^2_j+\omega_d^2-j^2\omega^2}\ ,\ \ \varphi = \tan^{-1}\frac{2\omega_d\omega'_j}{\omega_d^2-\omega'^2_j+j^2\omega^2},\ \alpha = \varphi + \tan^{-1}\frac{2\omega_d\omega'_j}{\omega'^2_j-\omega_d^2} \tag{25}$$

The dynamic tensile stress on the outer fibre of the I-Beam is obtained by:

$$\sigma_{dynamic\ ,t} = \frac{M_{x,t}\bar{y}}{I}\ \bar{y}\ \ is\ the\ distance\ to\ the\ outer\ fibre\ from\ the\ neutral\ axis \tag{26}$$

I is the second moment of area. When the train traverses the bridge, this results in the initial deflection of the bridge and the oscillations then occur about a mean position. As the fatigue S-N curves are given for zero mean stress conditions then the Goodman's relationship is used to take into account the effect of mean stress. Using a rain-flow counting algorithm within MATLAB the stress time histories are converted into a stress-range time history. This operation also gives the mean stress for each cycle count. The stress after Goodman correction is given by,

$$\sigma_{e,i,static\ /dynamic} = \frac{\sigma_{r,i,qstatic\ \ /dynamic}}{1-\left(\frac{\sigma_{m,i}}{\sigma_{UTS}}\right)} \tag{27}$$

Where, $\sigma_{m,i}$ is the mean stress and σ_{UTS} the ultimate tensile strength of the beam. The number of cycles to failure can then be calculated from equation (28), where m is the slope of the region of the S-N curve and depends on the stress range. Where the stress range is $>\sigma_o m=3.5$, and for stress range $<\sigma_o$, slope = $m+2$. The stress range, σ_o, is the constant amplitude fatigue limit of the detail under consideration.

$$Log(N_{f,i,qstatic\ /dynamic}) = Log(K_o) + dLog(\Delta) - mLog(\sigma_{e,i,qstatic\ /dynamic}) \tag{28}$$

For this assessment BS-5400 fatigue Class C is used for damage calculation with the following parameters,

K_o	$= 1.08\times10^{14}$	Constant term relating to the mean-line of the statistical analysis results
m	= 3.5	Inverse slope of the mean-line log $\sigma_r - Log(N)$ curve
Δ	= 0.625	The reciprocal of the anti-log of the standard deviation of $Log(N)$
d	= 2	The number of standard deviations below the mean-line

For Class C, σ_o= 78.2MPa.The cumulative damage index (CDI) is now calculated using Miner's rule.

$$CDI_{qstatic\ /dynamic} = \sum_{i=1}^{acc}\frac{n_{i,qstatic\ \ /dynamic}}{N_{f,i,qstatic\ \ /dynamic}} \tag{29}$$

A fatigue failure is deemed to occur when the value of CDI ≥ 1.0. The fatigue life, based on a total annual cycle count*(acc)* is given by the reciprocal of equation (29) where fatigue life in years = 1/CDI.

4 MOVING LOAD DYNAMIC ANALYSIS OF A PLATE GIRDER BRIDGE

4.1 *Introduction*

The analytical model described in the previous section was implemented within MATLAB. The model facilitates the addition of any number of bridges and different train types in a standard format. The standard set of train mixes defined in BS 5400 (1980) were included within the model, enabling the selection of any one type of train for assessment. The code defines a standard set of mixes for fatigue assessment; Light, Medium and Heavy traffic types. For this assessment the medium traffic type is used which consists of Train Nos. 1, 5, 7 and 8 with total annual frequencies as highlighted in Table 1.

The model only considers the primary vertical bending mode of the bridge, which in most cases, is the most critical mode for fatigue assessment. This simplifies the bridge input data to only providing the span (L),Young's Modulus (E) and Second Moment of Area (I). A damping ratio(ξ) may also be inputted if this information is available, otherwise damping is calculated using the empirical method given by Fryba (1999).

Table 1. BS-5400 (1980) standard train mixes.

Train No.	BS-5400 Train Type	Locomotive	Locomotive Axle Weight	Wagon Type	Wagon Axle Weights	No. of Wagons	Total Weight	Annual Frequency
1	Steel Train		6 x 21.5t		6 x 18.5t	15	1,794t	2,257
5	Diesel Hauled Passenger Train		6 x 20t		4 x 10t	12	600t	22,500
7	Heavy Freight		6 x 20t		4 x 25t	10	1,120t	2,411
8	Heavy Freight		6 x 20t		2 x 25t	20	1,120t	6,027

4.2 *Candidate Half-Through Plate Girder Bridge*

The case study bridge selected for fatigue damage assessment is an 18.1m span half-through plate girder railway bridge as shown in Figure 3(Gaillard, 2003). The bridge is constructed with two main longitudinal steel girders and steel transverse cross-girders encased in concrete fill. The transverse girders bear onto the main girder bottom flanges. The bridge supports a single, centrally located track and has a span which is representative of typical medium span bridges on the UK railway network.

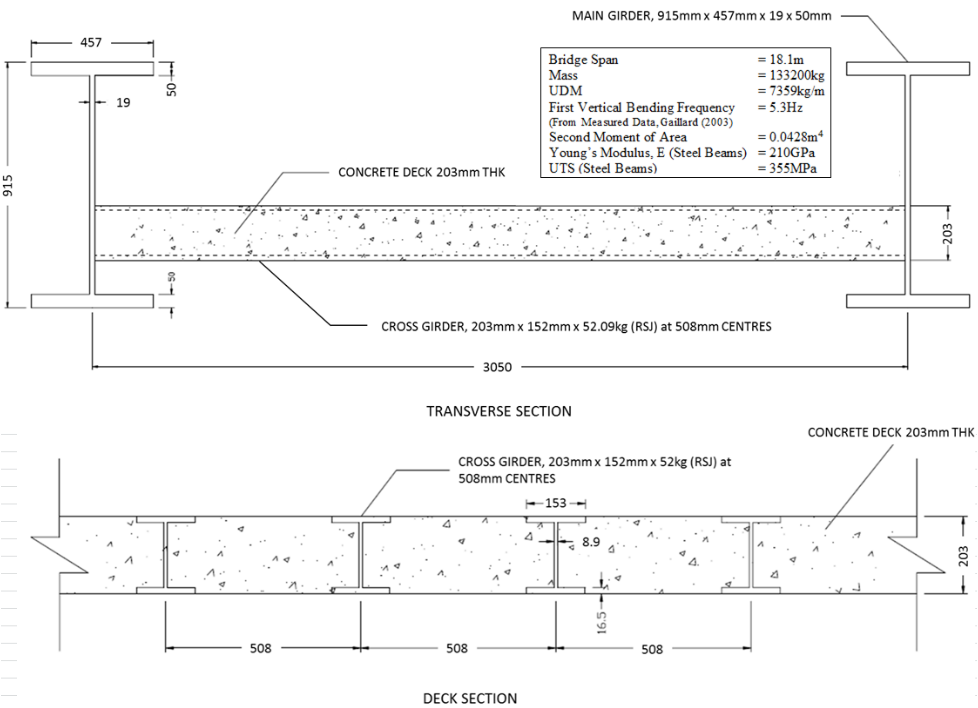

Figure 3. Half-through deck plate girder railway bridge used as case study (Gaillard, 2003).

4.3 *Bridge Response Results & Discussion*

The stresses are calculated on the outer fibre of the main longitudinal girders and the response for each train type is shown in Figure 4. To account for dynamic effects in the quasi-static (q-static) analysis the static stresses have been multiplied with the calculated Dynamic Amplification Factor (DAF) 1.065 at 50km/h. The results show that peak responses are marginally higher than for the dynamic case Train 1, indicating that the use of the DAF results in a more conservative result for this train. When the locomotive axle loads are higher than the wagon axle loads, typical of passenger trains, this results in the first stress peak being higher as shown in Figure 4 for Trains 5 and 8. It is interesting to note that for Train 8, which has two-axle wagons following the locomotive, the stress variation is relatively lower as compared to the other trains.

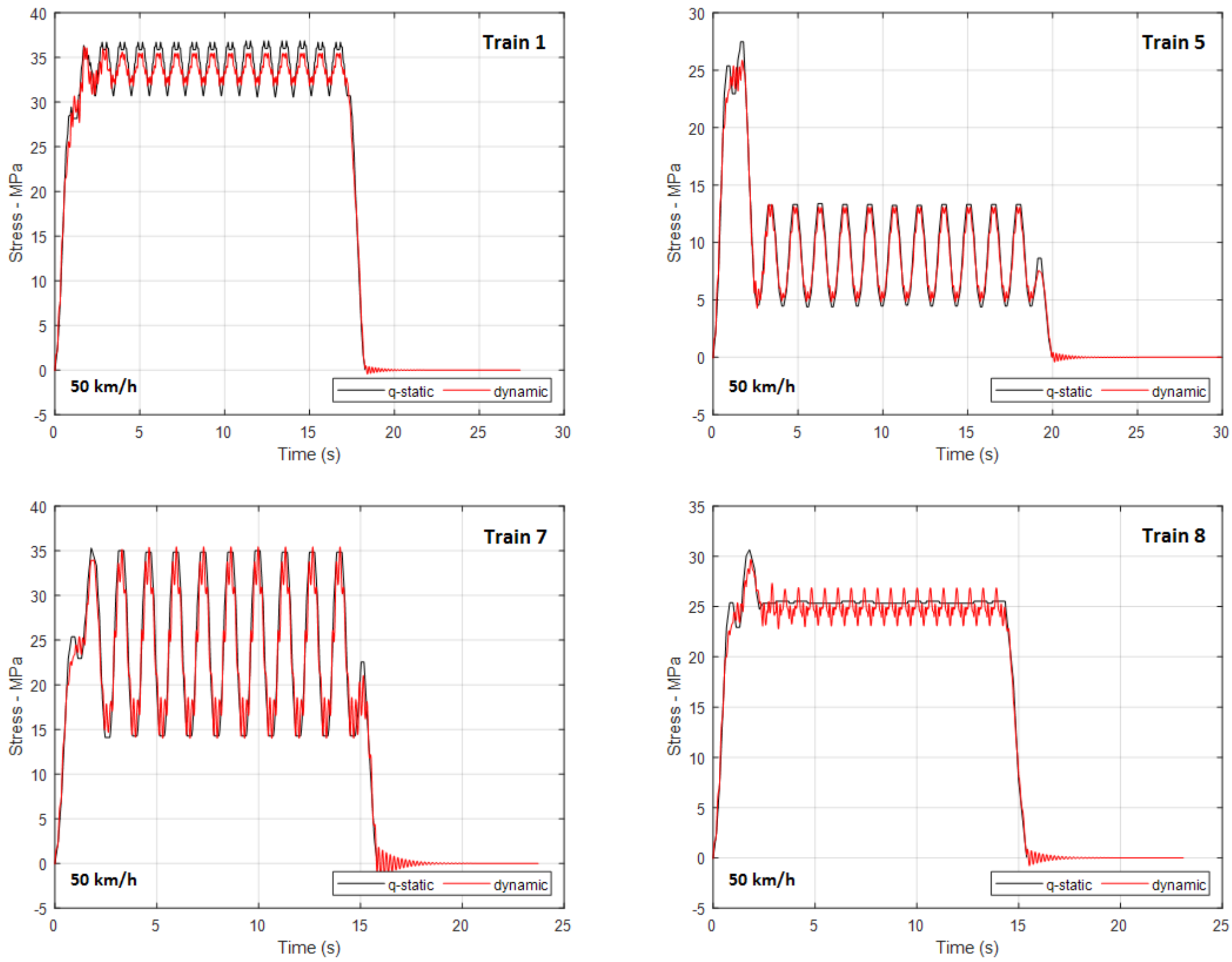

Figure 4. Bridge stress-time response for different trains comparing static versus dynamic analyses.

Using the Rainflow technique in MATLAB the stress-time histories of Figure 4 are converted into stress range histograms which are then used to perform the Miner's Cumulative Damage Index calculations. The results of the Cumulative Damage Index (CDI) calculations for train speeds up to 200km/h for each train type are shown in Figure 5. As expected, the CDI for purely a static loading case, where no DAF is applied results in a nearly constant CDI throughout the speed range. When the DAF is applied to the static stresses then the CDIs increase with speed, but no resonance effects are observed as this analysis does not capture the dynamic response of the bridge explicitly. For the dynamic case, the results show that the CDIs are generally lower than their quasi-static counterparts, except when the condition of resonance occurs, which are indicated by the spikes in the CDI curves. For Trains 1 and 5 resonance conditions at 170km/h and 190km/h are apparent. Train 7 indicates two resonance conditions, one at 120km/h and the other at 180km/h. Train 8 shows a resonance condition at 170km/h. At train speeds where resonance conditions may occur the CDI values are shown to be significantly higher as compared to other speeds.

Figure 6 shows the response plots for Train 7 at speeds which cause a resonance condition, clearly showing considerable amplification of the stress levels. The quasi-static response using a DAF is incapable of capturing resonance effects thus leading to potential under-estimation of fatigue life.

A summary of the Cumulative Damage Index and fatigue life for the Medium Traffic load model consisting of Trains 1, 5, 7 and 8 are shown in Table 2 for both the quasi-static (top) and dynamic (bottom) analysis. The results are given for three speeds, 50 km/h, 70 km/h and 120 km/h. For both analyses, the fatigue life is can be seen to reduce for increasing speeds. At 50 km/h the fatigue life obtained by dynamic analysis is found to be 8.7% higher than that of an equivalent quasi-static analysis. Similarly, at 70km/h the fatigue life is found to be higher by 18% given by the dynamic analysis. However, if trains are operated at conditions of resonance then there is a significant reduction in the fatigue life. At 120 km/h, it can be seen that the fatigue life given by the dynamic analysis is significant lower (40%) than its quasi-static counterpart clearly demonstrating the inability of DAFs capturing such resonance dynamic effects on fatigue damage.

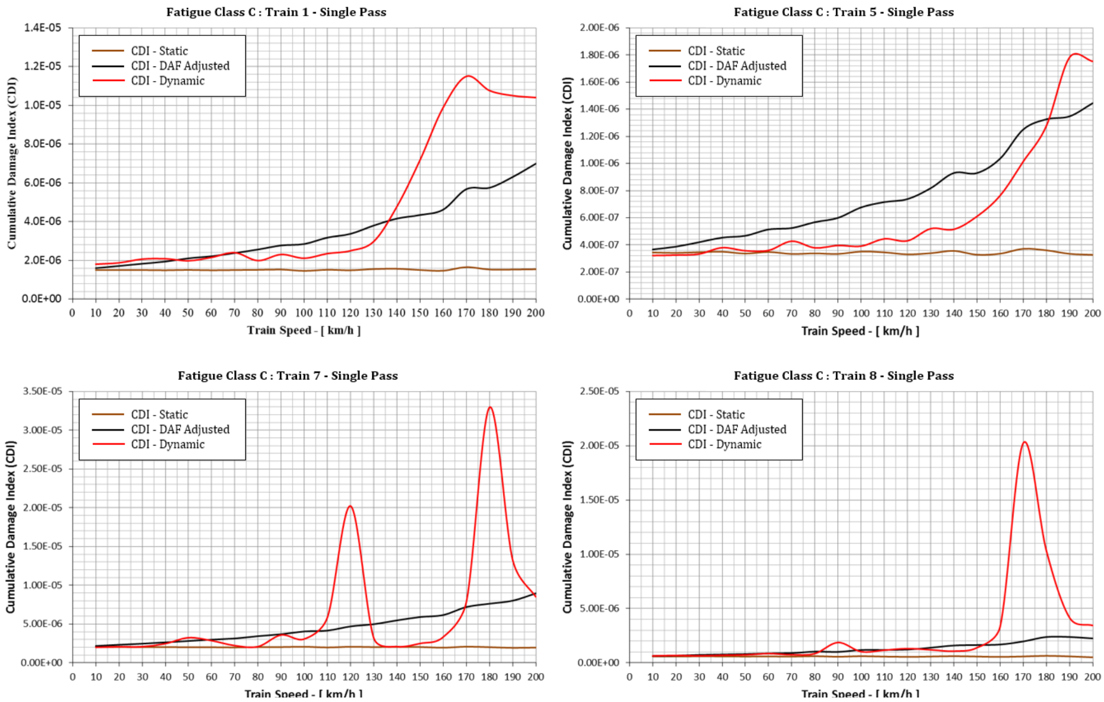

Figure 5. Bridge cumulative fatigue damage vs train speed for single train passages of 4 medium traffic trains.

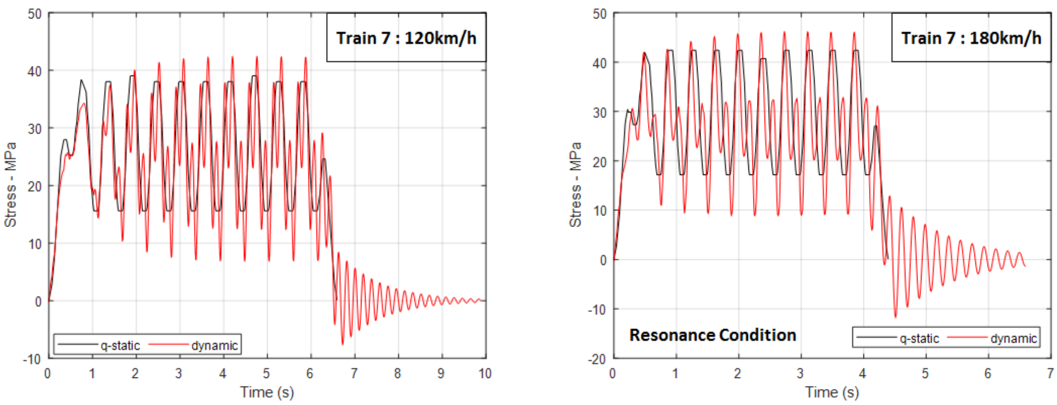

Figure 6. Bridge stress response at resonance conditions for Train 7.

Table 2. Comparison of fatigue cumulative damage index between quasi-static and dynamic analyses.

Quasi-Static Analysis (DAF Multiplied)			**50 km/h**		**70 km/h**		**120 km/h**	
Train No.	BS-5400 Train Type	Annual Frequency	Single Pass CDI	Annual CDI	Single Pass CDI	Annual CDI	Single Pass CDI	Annual CDI
1	Steel Train	2257	2.11E-06	0.004754	2.38E-06	0.005373	3.37E-06	0.00761
5	Diesel Hauled Passenger Train	22500	4.67E-07	0.010511	5.24E-07	0.011798	7.38E-07	0.016603
7	Heavy Freight Train	2411	2.81E-06	0.006777	3.14E-06	0.007566	4.69E-06	0.011314
8	Heavy Freight Train	6027	7.96E-07	0.0048	8.92E-07	0.005374	1.22E-06	0.007332
Total Annual CDI For Medium Traffic				0.02684		0.03011		0.04286
Fatigue Life (Years)				37.3		33.2		23.3
Dynamic Analysis (Euler-Bernoulli Beam Model)			**50 km/h**		**70 km/h**		**120 km/h**	
Train No.	BS-5400 Train Type	Annual Frequency	Single Pass CDI	Annual CDI	Single Pass CDI	Annual CDI	Single Pass CDI	Annual CDI
1	Steel Train	2257	1.98E-06	0.004473	2.4E-06	0.005415	2.5E-06	0.005639
5	Diesel Hauled Passenger Train	22500	3.57E-07	0.008032	4.27E-07	0.009617	4.3E-07	0.009676
7	Heavy Freight Train	2411	3.24E-06	0.007803	2.21E-06	0.005338	2.02E-05	0.048758
8	Heavy Freight Train	6027	7.29E-07	0.004392	8.43E-07	0.005081	1.31E-06	0.007872
Total Annual CDI For Medium Traffic				0.02470		0.02545		0.07195
Fatigue Life (Years)				40.5		39.3		13.9
Fatigue Life Amplification Factor				**1.09**		**1.18**		**0.60**
Fatigue Life Increase/Decrease				**8.67%**		**18.31%**		**-40.43%**

5 CONCLUSIONS

In this paper an analytical model, using the Euler-Bernoulli Beam (EBB) formulation, has been presented for estimating the fatigue damage accumulation on a plate girder railway bridge due to the dynamic interaction effects of the bridge and train. The model has been presented in a generalised form and implemented within MATLAB, enabling a wide range of bridge and train configurations to be assessed. A single span plate girder bridge was used to estimate the damage accumulation using the Medium Train Mix defined in BS-5400 (1980), as a case study. Results have also been presented for a quasi-static based analysis, whereby the static stress-time history is multiplied by the code-estimated DAF. The main conclusions from this study are as follows.

- In the quasi-static analysis (DAF applied), fatigue damage accumulation increases with speed and is generally higher than that estimated through a dynamic analysis when speeds are not close to a resonance.
- Dynamic analyses showed that at resonance conditions significant fatigue damage accumulation can result. For this particular case study bridge, for train speeds of 120km/h the fatigue life estimated is 40% lower than the equivalent damage estimated through the use of DAFs, due to resonance effects of Train 7.
- Where trains are not operating at or near any resonance conditions, the dynamic analysis results in longer fatigue life estimates as compared to quasi-static analysis using a DAF. At 50km/h and 70km/h the dynamic analysis results in 8.7% and 18.3% higher fatigue lives, respectively.

The results of this study have shown that the application of DAFs on static stress histories can be overly conservative giving a lower fatigue life estimate. The dynamic analysis has shown that at resonance conditions significant reductions in fatigue life can be expected. However, away from this condition, the dynamic analysis is shown to give higher life estimates than for a quasi-static type analysis. The results from this study can be useful for bridge asset owners for assessment and management of bridges assets, giving them the means to identify optimal train speeds for given train configurations to maximise fatigue life.

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